Discuss the continuity of each function

(a) $f(x) = \frac{1}{x}$

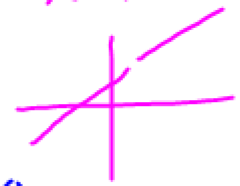
infinite discontinuity at $x = 0$, or on any interval surrounding $x = 0$

A continuous function

continuous on interval $(-\infty, 0) \cup (0, \infty)$

(b) $g(x) = \frac{x^2 - 1}{x - 1}$

$x = 1$ not domain, removable discontinuity



continuous on interval

$(-\infty, 1) \cup (1, \infty)$

continuous function

(c) $h(x) = \begin{cases} x + 1, & x \leq 0 \\ x^2 + 1, & x > 0 \end{cases}$

continuous function, no discontinuity

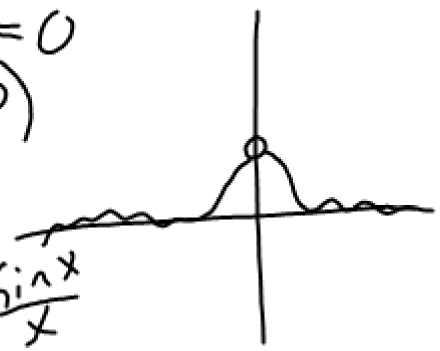
(d) $y = \frac{\sin x}{x}$

removable discontinuity at $x = 0$

continuous $(-\infty, 0) \cup (0, \infty)$

$y = 1$ at $x = 0$

$-\infty < x < 0 \quad 0 < x < \infty$



$\lim_{x \to 0} \frac{\sin x}{x}$

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Greatest integer function

$f(x) = [\![x]\!]$ greatest integer $\leq x$

$f(x) = \text{int } x$

$\text{int}(1.1) = 1$

$\text{int}(3.7) = 3$

$\text{int}(-1.8) = -2$

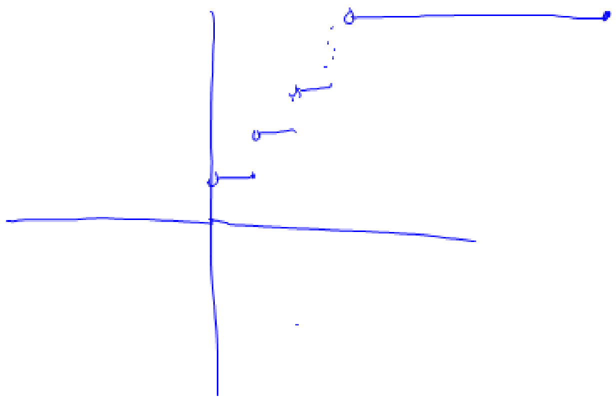
$f(x) = -\text{int}(-x)$

$f(x) = -\text{int}\left(-\frac{1}{2}\right)$

$-1 = 1$

$7.25 \div 1.1 = 6.59$

$$f(x) = \begin{cases} -1.10\,\text{int}(-x), & x \leq 6 \\ 7.25, & 6 < x \leq 24 \end{cases}$$



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$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 1 & x = 1 \\ -x+3 & 1 < x < 2 \end{cases}$$

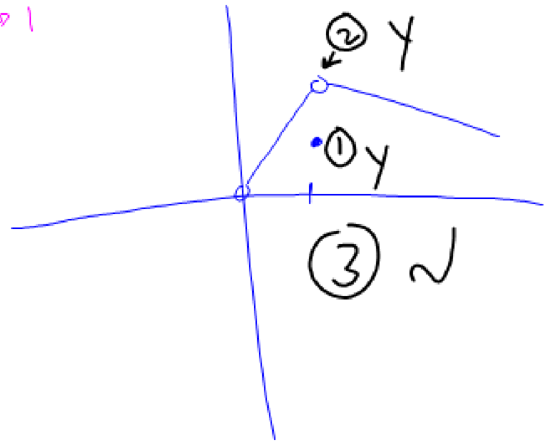
$f(1)$ doesn't exist F

$\lim_{x \to 0^+} f(x)$ exists T

$\lim_{x \to 2^-} f(x)$ exist T

$\lim_{x \to 1} f(x)$ exist T

$\lim_{x \to 1} f(x) = f(1)$ F



continuity at a point, c

① $f(c)$ exists

② $\lim_{x \to c} f(x)$ exists

③ $\lim_{x \to c} f(x) = f(c)$

① Find the avg. rate of change of $f(x)=x^3-x$ over the interval $[1,3]$.

$f(1) = 1^3 - 1 = 0$

$f(3) = 3^3 - 3 = 24$

$(1,0) \quad (3,24)$

$m = \dfrac{24-0}{3-1} = 12$

Slope secant line

② Find the rate of change at $x=2$.

$f(x) = x^3 - x$

$f(x+h) = (x+h)^3 - (x+h) \qquad -(x^3 - x)$

$$\frac{\cancel{x^3} + 3x^2\cancel{h} + 3h^{\cancel{2}}x + h^{\cancel{3}} \cancel{- x} - \cancel{h} - \cancel{x^3} \cancel{+ x}}{\cancel{x} + \cancel{h} \cancel{- x}}$$

$\lim_{h \to 0} \; 3x^2 + \cancel{3hx} + \cancel{h^2} - 1 = 3x^2 - 1$

$= 3(2)^2 - 1 = 11$

(3) What is the rate of change of the volume of a sphere ($V = \frac{4}{3}\pi r^3$) with respect to the radius when the radius is 3 in?

Do Sect. 2.3 #18, 21, 24, 33, 42 + 5 others

Read 2.4 work through examples

Volume prob

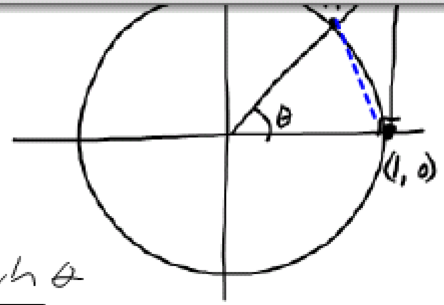
$3 \le 5$

$\frac{1}{3} \ge \frac{1}{5}$

$\tan = \frac{\sin\theta}{\cos\theta}$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\cancel{2}} \cdot \frac{\cancel{2}}{\sin\theta}$$

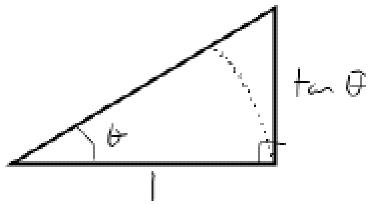
$$\frac{1}{\cos\theta}$$



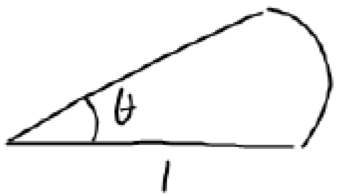
$A(\cos\theta, \sin\theta)$

$B(1, \tan\theta)$

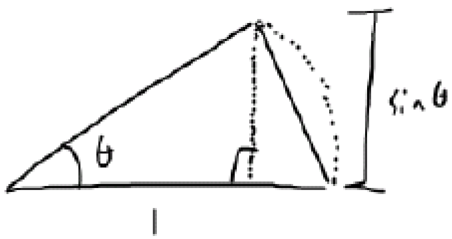
Area



$\frac{\tan\theta}{2}$



$$\frac{\theta}{2} \quad \left[\text{area sector } \frac{\theta}{2\pi}(\pi r^2) = \frac{r^2\theta}{2}\right]$$



$\frac{\sin\theta}{2}$

$$\frac{\sin\theta}{2} \le \frac{\theta}{2} \le \frac{\tan\theta}{2}$$

mult by $\frac{2}{\sin\theta}$

$$1 \le \frac{\theta}{\sin\theta} \le \frac{1}{\cos\theta}$$

Take reciprocal, switch:

$$1 \ge \frac{\sin\theta}{\theta} \ge \cos\theta$$

use Sandwich Theorem

take limit

$$\lim_{x\to 0}\left(1 \ge \frac{\sin\theta}{\theta} \ge \cos\theta\right)$$

$$\lim_{x\to 0} 1 \ge \lim_{x\to 0} \frac{\sin\theta}{\theta} \ge \lim_{x\to 0} \cos$$

$\downarrow$